D-I(Hons.)

Date - 25-03-2020

$c' = c - v$

(Clearly a different transformation is required if the postulates of special relativity are to be satisfied)

Lorentz transformation:

* The correct relationship b/w x and x' is,

$$x' = k(x - vt) \quad \text{—— (8)}$$

Here, k is a factor that does not depend either x or t but may be a function of v.

The choice of eqⁿ (8), follows from several considerations:

1. It is linear in x and x', so that a single event in frame S corresponds to a single event in frame S'.

2. It is simple;
3. It has the possibility of reducing to eqⁿ (1)

eqs. of physics must have the same form in both S and S', the corresponding eq for x in terms of x' and t':

$$x = k(x' + vt') \quad \text{—— (9)}$$

There is no difference b/w y, y' and z, z' which are ⊥ to the direction of v. Hence,

$$y' = y \quad \text{—— (10)}$$

$$z' = z \quad \text{—— (11)}$$

the time co-ordinates t and t' are not equal. It can be seen, by substituting the value of x' given by eqn (8) into eqn (9). This gives

$$x = k\left[k(x-vt) + vt'\right]$$

$$x = k^2(x-vt) + kvt'$$

from which we find,

$$t' = kt + \left(\frac{1-k^2}{kv}\right)x \qquad (12)$$

eqn (8) and (10) to (12) Constitute a co-ordinate transformation that satisfies the first postulate of special relativity.

At t=0, the origins of the two frames of reference S and S' are in the same place, and $t'=0$. Suppose, that a flare is set off at the common origin of S and S' at $t=t'=0$ and the observers in each system measure the speed with which the flare's light spreads out. Both Observers must find the same speed c, which means that in the S fram:

$$x = ct \qquad (13)$$

and in the S' frame

$$x' = ct' \qquad (14)$$

Substituting for x' and t' in eqn (14),

$$k(x-vt) = ckt + c\left(\frac{1-k^2}{kv}\right)x$$

and solving for x,

$$x = \frac{ckt + vkt}{k - \left(\frac{1-k^2}{kv}\right) \cdot c}$$

$$x = ct\left[\frac{k + \frac{v}{c}k}{k - \left(\frac{1-k^2}{kv}\right)c}\right] = ct\left[\frac{1+\frac{v}{c}}{1-\left(\frac{1}{k^2}-1\right)\frac{c}{v}}\right]$$

the above expression will be same as, eq^n (B), If

$$\frac{1+\frac{v}{c}}{1-\left(\frac{1}{k^2}-1\right)\frac{c}{v}} = 1$$

and,

$$k = \frac{1}{\sqrt{1 - v^2/c^2}} \quad \text{——— (15)}$$

~~using eq^n (15)~~ putting the value of k in eq^n (1), (2) & (3)

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}} \quad \text{——— (16)}$$

$$y' = y \quad \text{——— (17)}$$

$$z' = z \quad \text{——— (18)}$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{——} \quad (19)$$

these equations (16) to (19) Comprise the Lorentz transformation. Lorentz transformation reduces to the Galilean transformation when the relative velocity v is small compared with the velocity of light c.